ISI – Bangalore Center – B Math - Physics IV – Mid Semester Exam
Date: 22 February 2016. Duration of Exam: 3 hours
Total marks: 50

In all questions units chosen are such that c=1. The Greek indices μ,ν take values 0,1,2,3 and the Latin indices *i,j* take values 1,2,3. The contravariant position four vector is given by $x^\mu = (x^0 = ct = t, x^1 = x, x^2 = y, x^3 = z)$ and the Lorentz transformation is represented by the matrix L^μ_ν where μ,ν represent the rows and columns respectively. All Lorentz transformations are proper.

Q1. [Total Marks: 4+6+3=12]

a.) Show that the sign of the time component of a space like vector can be changed (from positive to negative and vice versa) under a suitable Lorentz transformation.

b.) Let two rods AB and CD of equal rest length L be initially far apart and move towards each other along the x axis. . The rod AB sees CD moving towards AB along its positive x axis with velocity v (>0). Let S be the frame in which AB is at rest; the space-time coordinates of A and B in this frame are (t,0,0,0) and (t,L,0,0). Let S' be the frame in which CD is at rest; the coordinates of C and D in this frame are (t', -L,0,0) and (t', 0,0,0). The clocks are set so that when A and D meet the time is zero in both frames.

In the frame S, the rest frame of AB

Fill the blanks the following table:

Coordinates of event	In S	In S'
When A and D meet: x^μ_{AD}	(0,0,0,0)	(0,0,0,0)
When A and C meet: x_{AC}	(--, --, 0,0)	(--, --, 0,0)
When B and D meet: x^μ_{AD}	(--, --, 0,0)	(--, --, 0,0)
When B and C meet: x^μ_{AD}	(--, --, 0,0)	(--, --, 0,0)

c.) Show that there exists a frame in which the events “when A and C meet” and “when B and D meet” are simultaneous.

Q2. [Total Marks: 4+2+4=10]

a.) Let A^μ, B^μ be two non zero four vectors. A^μ is time like, and $A.B = g_{\mu\nu}A^\mu B^\nu = 0$. Determine if B^μ is time like, space like, light like or may not be any of the above.

b.) Let U^μ be the velocity four vector defined by $U^\mu = \dfrac{dx^\mu}{ds}$, where ds is the invariant interval. Show that U^μ is a unit time like vector.

c.) The acceleration four vector is defined by $a^\mu = \dfrac{dU^\mu}{ds}$. Determine if a^μ is time like, space like, light like or may not be any of the above.

Q3. [Total Marks: 6+4=10]

a.) Let there be n particles with momentum four vector $p_1^\mu, p_2^\mu, ..., p_n^\mu$ with $p_\alpha^\mu = (p_\alpha^0 = m_\alpha \gamma_{u_\alpha}, p_\alpha^i = m_\alpha \gamma_{u_\alpha} u_\alpha^i)$. Here α denotes the α-th particle with rest mass m_α with speed u_α^i. Define $P_{tot}^\mu = \sum_\alpha p_\alpha^\mu$. Show that there exists a frame, called the CM frame, in which $\vec{P}_{tot} = 0$. State clearly the theorems you may be using to prove this result.

b.) A particle of rest mass *m*, and energy *E* collides with an identical particle at rest. The collision produces two more of the same particles (i.e. after the collision there are 4 identical particles with the same rest mass *m*). Assuming conservation of linear four momenta, show that *E* must be greater than 7*m*.
[Hint: first calculate the total energy in the CM frame in terms of *E*]

Q4. [Total Marks: 2+2+4=8]

a.) Let L_1 and L_2 be two Lorentz Transformations (LT) representing two arbitrary boosts. Using properties of boost transformations, explain why $L_2 L_1$ is in general a boost followed by a rotation. (your answer should be brief).

b.) Let L_ν^μ be a LT representing a rotation. What are the values of L_ν^0 ?

c.) Let the rotation in part b.) above be represented by a 3x3 matrix R_j^i. Let $F^{\mu\nu}$ be the antisymmetric electromagnetic field tensor with $F^{0i} = -E^i, F^{ij} = -\varepsilon^{ijk} B^k$. Show that under a LT that is a rotation the electric fields transform as a ordinary 3-vectors.

Q 5. [Total Marks: 5+5=10]

Let a particle be a subject to a constant force $\vec{F} = (m_0 g, 0, 0)$ where m_0 is the rest mass of the particle.

a.) Let the initial speed of the particle be zero. Show that at subsequent times the components of velocity in y and z directions continue to be zero. Determine the speed of the particle as a function of *t* and show that it never exceeds the speed of light.

b.) Let the components of initial velocity of the particle be zero in the x and z direction but non zero in the y direction. Show that the ratio of the velocity component in the x direction to the velocity component in the y component is a function of time as given by $\frac{gt}{\gamma(u_{initial})u_{initial}}$ where $u_{initial}$ is the initial speed